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TOWARDS A SUPERSYMMETRIC NON-ABELIAN BORN-INFELD THEORY

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We define an iterative procedure to obtain a non-abelian generalization of the Born-Infeld action. This construction is made possible by the use of the severe restrictions imposed by kappa-symmetry. We have calculated all bosonic terms in the action up to terms quartic in the Yang-Mills field strength and all fermion bilinear terms up to terms cubic in the field strength. Already at this order the fermionic terms do not satisfy the symmetric trace-prescription.

1. Abelian Born-Infeld

One of the most intriguing features of D-branes is their close connection to gauge theories. Indeed, the effective theory describing the world-volume dynamics of a Dp-brane is a $p+1$–dimensional field theory with, in the static gauge, as bosonic degrees of freedom the transversal coordinates X^m $(m = 1, \cdots, 9-p)$ of the brane, and the massless states of the open strings ending on the brane which appear as a $U(1)$ gauge field describing $p-1$ degrees of freedom. When these fields vary slowly, the effective action governing their dynamics is known to all orders in α'. It is the ten-dimensional Born-Infeld action,[1] dimensionally reduced to $p+1$ dimensions. In the supersymmetric case there are additional fermionic degrees of freedom χ_α $(\alpha = 1, \cdots 16)$ describing 8 fermionic degrees of freedom so that we have a total of $8+8$ degrees of freedom.

Clearly, the $9-p$ transversal scalars break the 10-dimensional Lorentz covariance. The fully covariant worldvolume theory of a single D-brane in a type II theory can be formulated in terms of the following worldvolume fields:[a] the embedding coordinates $X^\mu(\sigma)$ (of which only the transverse coordinates represent physical degrees of freedom), the Born-Infeld vector field $V_i(\sigma)$, and $N=2$ spacetime fermionic fields $\theta(\sigma)$. The presence of a local fermionic symmetry (κ-symmetry) makes it possible

[a] The indices $\mu, \nu = 0, \ldots, 9$ are spacetime indices whereas the indices $i, j = 0, \ldots, p$ label the worldvolume coordinates σ^i.

to gauge away half the fermionic degrees of freedom. The field content then corresponds, in a static gauge, to that of a supersymmetric Yang-Mills theory in $p+1$ dimensions describing $8+8$ degrees of freedom.

In this talk our main emphasis will be on the IIB D9–brane. This case is special in the sense that, for $p = 9$, there are no transverse scalars and the worldvolume theory is given by a D=10 supersymmetric Maxwell multiplet (V_μ, χ). The bosonic part of the action is given by the ten-dimensional Born-Infeld Lagrangian[2,3]

$$\mathcal{L}(\text{bosonic}) = -\sqrt{-\det\left(\eta_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}\right)}\ . \tag{1}$$

Expanding the square root this action gives rise to an infinite number of terms with even powers of F. The term quadratic in F is the usual Maxwell kinetic term.

In the supersymmetric case every F^n term gets a supersymmetric partner and the complete action has the schematic form

$$\begin{aligned}\mathcal{L}(\text{supersymmetric}) &= \operatorname{tr}F^2 + (2\pi\alpha')^2\left[\operatorname{tr}F^4 - \tfrac{1}{4}\left(\operatorname{tr}F^2\right)^2\right] + \cdots \\ &+ \bar{\chi}\partial\chi + (2\pi\alpha')^2 F^2\bar{\chi}\partial\chi + \cdots\end{aligned}$$

The fermionic $F^2\bar{\chi}\partial\chi$ terms were found in Refs. 4,5 from the requirement that the action must possess a linear supersymmetry. Complete results were obtained in D=4 dimensions via superspace techniques.[6]

Recently a new development took place which led to a construction of the complete supersymmetric Born-Infeld action. A crucial role in this development was played by kappa-symmetry. The kappa-symmetric covariant D-brane actions were constructed in flat,[7] as well as in curved backgrounds.[8,9]

Schematically, the kappa-symmetric Lagrangian is given by[b]

$$\mathcal{L}_{\mathrm{BI}} = -e^{-\phi}\sqrt{-\det\left(g + \mathcal{F}\right)}\ +\ Ce^{\mathcal{F}}\,, \tag{2}$$

with $\mathcal{F} = 2dV + B$. The first term is often called the kinetic term whereas the second one is the Wess-Zumino term. It is understood that all NS-NS background fields g, ϕ, B and all R-R background fields C are superfields defined over a superspace with coordinates (X^μ, θ). The local κ-symmetry acts on the fermions as

$$\delta\bar{\theta}(\sigma) = \bar{\eta}(\sigma) \equiv \bar{\kappa}(\sigma)\,(1+\Gamma)\,, \tag{3}$$

where Γ, which depends on worldvolume as well as background fields, satisfies

$$\Gamma^2 = \mathbb{1}\,. \tag{4}$$

The projection (3) makes it possible to gauge away half the fermionic degrees of freedom. The variation of the D-brane action takes the form

$$\delta\mathcal{L} = -\bar{\eta}\,(1-\Gamma)\mathcal{T}\,, \tag{5}$$

[b]From now on we set $2\pi\alpha' = 1$.

where $\mathcal{T}$ is some expression in terms of the worldvolume and background fields. The variation (5) indeed vanishes if $\bar{\eta}$ is given by (3). This variation has the following source: the first term within the round brackets comes from the kinetic term in (2), the term with Γ arises from the Wess-Zumino contribution.

From the above it is clear that a crucial role is played by the projection operator $1+\Gamma(X,\mathcal{F})$. In the case of the IIB D9–brane the $\mathcal{F}$-independent part $\Gamma^{(0)}(X)$ of $\Gamma(X,\mathcal{F})$ is given by

$$\Gamma^{(0)}(X) = \frac{1}{10!\sqrt{-\det g}}\epsilon^{i_1\dots i_{10}}\gamma_{i_1\dots i_{10}}\,, \qquad \left(\Gamma^{(0)}(X)\right)^2 = \mathbb{1}. \tag{6}$$

The worldvolume metric reads

$$g_{ij} = \eta_{ij} + \bar{\theta}\gamma_{(i}\partial_{j)}\theta\,, \qquad \eta_{ij} \equiv \partial_i X^\mu \partial_j X_\mu\,, \tag{7}$$

and

$$\gamma_i \equiv \Gamma^\mu \partial_i X^\mu\,. \tag{8}$$

The complete expression for $\Gamma(X,\mathcal{F})$ is given by[9]

$$\Gamma = \frac{\sqrt{-\det g}}{\sqrt{-\det(g+\mathcal{F})}}\Gamma^{(0)}\sum_{k=0}^{5}\frac{1}{2^k k!}\mathcal{P}_{(k)}\gamma^{i_1\dots i_{2k}}\mathcal{F}^k_{i_1\dots i_{2k}}\,, \tag{9}$$

where

$$\mathcal{P}_{(k)} = \sigma_1 \text{ (for } k=1,3,5)\,, \quad \mathcal{P}_{(k)} = i\sigma_2 \text{ (for } k=2,4)\,. \tag{10}$$

The Pauli-matrices refer to the fact that we are using an N=2 superspace notation.

It was realized in Ref. 7 that for the special case of a IIB D9–brane in a flat background, the results take on a particularly simple form. We consider a flat background, i.e. we take $e_\mu{}^a = \delta_\mu{}^a$ and all other fields zero. Furthermore, we fix the kappa-transformations and worldvolume parametrizations (we choose the static gauge) by setting

$$X^\mu = \delta^{i\mu}\sigma^i\,, \qquad \theta = \begin{pmatrix}\theta_1 \equiv \chi \\ \theta_2 = 0\end{pmatrix}. \tag{11}$$

The result of this gauge-fixing is that the complete Wess-Zumino term vanishes and the kinetic term is given by the simple expression[7]

$$\mathcal{L}_{\rm BI} = -\sqrt{-\det\left(\eta_{\mu\nu} + F_{\mu\nu} + \bar{\chi}\Gamma_\mu\partial_\nu\chi + \tfrac{1}{4}\bar{\chi}\Gamma^a\partial_\mu\chi\bar{\chi}\Gamma_a\partial_\nu\chi\right)}\,. \tag{12}$$

This action has 16 linear supersymmetries as well as 16 nonlinear supersymmetries. The explicit form of these supersymmetry rules can be obtained from the original 32 supersymmetries of the N=2 superspace after taking care of the fact that they get deformed with a field-dependent kappa-transformation upon fixing the kappa-gauge.

Our goals, which we will discuss in this talk, are two-fold:

(1) We want to find the non-abelian version of the projection operator $1 + \Gamma$. One application we have in mind is that knowing the explicit expression of Γ enables us to investigate non-abelian BPS configurations. The condition of supersymmetry for such configurations is given by $(1 - \Gamma)\epsilon = 0$.[10]

(2) A second, and more ambitious goal is to construct the non-abelian supersymmetric Born-Infeld action by the use of the severe restrictions of kappa-symmetry as suggested in Ref. 11. We restrict ourselves to a flat background for reasons to be discussed later (see the comments section). Non-abelian N=2, D=4 supersymmetric Born-Infeld actions (with a symmetrized trace condition) have recently been constructed in Refs. 12,13.

It should be stressed that we only discuss the IIB D9–brane. We expect that at least some of the structure of the lower-dimensional branes can be obtained by applying T-duality.[14,15] These lower-dimensional cases have the additional complication that they contain non-trivial transverse scalars. It is an open question what kind of "D-geometry" these scalars are supposed to describe.[16,17]

2. Non-Abelian Born-Infeld

In the case of a single D–brane the worldvolume theory, in lowest order in α' is given by a supersymmetric Maxwell theory. The result to all orders in α' is given by the supersymmetric BI action.

Once several D-branes are present, the situation changes. The mass of the strings stretching between two branes is proportional to the shortest distance between the two branes. Starting off with n well separated D-branes, we end up with a $U(1)^n$ theory. However, once the n branes coincide, additional massless states appear which complete the gauge multiplet to a non-abelian $U(n)$-theory.[18] Contrary to the abelian case, the effective action is not known to all orders in α'. The first term, quadratic in the field strength, is nothing but a dimensionally reduced $U(n)$ Yang-Mills theory. The next order, which is quartic in the field strength, was obtained from the four-gluon scattering amplitude in open superstring theory[19] and from a three-loop β-function calculation.[20]

The notion of an effective action for slowly varying fields is subtle in the non-abelian case.[1] In the effective action higher derivative terms are dropped. However because of

$$D_i D_j F_{kl} = \frac{1}{2}\{D_i, D_j\} F_{kl} - \frac{i}{2}[F_{ij}, F_{kl}], \tag{13}$$

this is ambiguous. The analysis of the mass spectrum seems to indicate that the symmetrized product of derivatives acting on a fieldstrength should be viewed as an acceleration term which can safely be neglected, while the anti-symmetrized products should be kept.

Different order prescriptions of the non-abelian matrices occurring in the non-abelian Born-Infeld action have been given in the literature. In Ref. 21 a symmetrized trace prescription was suggested: the non-abelian Born-Infeld action assumes essentially the same form as the abelian one, however, all Lie algebra valued objects have to be symmetrized first before taking the trace. Other trace prescriptions, involving commutators, have been given as well.[22]

Recently, it was found that the symmetrized trace prescription could not be correct as it did not reproduce the mass spectrum of certain D-brane configurations.[23,11] It was shown in Ref. 24 that by adding commutator terms to the action the problem might be cured.

For n overlapping D9-branes the completely gauge-fixed result should be the supersymmetric version of the non-abelian Born-Infeld theory. Since the vector fields $V_i^A(\sigma)$, $A = 1, \dots, n^2$, are in the adjoint representation of $U(n)$, we have to make the same choice for the fermion fields θ. Therefore we start out with fields $\theta^A(\sigma)$, which form a doublet ($N = 2$) of Majorana-Weyl spinors for each A, satisfying $\Gamma_{11}\theta^A = \theta^A$. After κ-gauge-fixing only half of each doublet will remain, and we have the correct number of degrees of freedom for the supersymmetric Yang-Mills theory.

This requires, that there are as many κ-symmetries as θ's, so that also the parameter of the κ transformations will have to be in the adjoint of $U(n)$. Thus the θ^A transform as follows under ordinary supersymmetry (ϵ), κ-symmetry (κ), Yang-Mills transformations (Λ^A), and worldvolume reparametrisations (ξ^i):

$$\delta\bar{\theta}^A(\sigma) = -\bar{\epsilon}^A + \bar{\eta}^A(\sigma) + f^A{}_{BC}\Lambda^B(\sigma)\bar{\theta}^C(\sigma) + \xi^i(\sigma)\partial_i\bar{\theta}^A(\sigma)\,, \tag{14}$$

with $\bar{\eta}^A \equiv \bar{\kappa}^B(\mathbb{1}\delta^{BA}+\Gamma^{BA})$. Here ϵ^A are constant, Γ^{AB} depends on the worldvolume fields, and therefore on σ. It must satisfy

$$\Gamma^{AB}\Gamma^{BC} = \delta^{AC}\mathbb{1}. \tag{15}$$

Useful information is obtained by considering commutators of these transformations. Because ϵ^A is constant we find from the commutator of Yang-Mills and supersymmetry transformations that

$$f^A{}_{BC}\Lambda^B\epsilon^C = 0 \to f_{ABC}\epsilon^C = 0\,. \tag{16}$$

Therefore $\epsilon = \epsilon^A T_A$, where T_A are the $U(n)$ generators, must be proportional to the unit matrix, i.e. we can choose a basis in which there is only one nonvanishing ϵ parameter. So only a subset of the θ^A transform under supersymmetry, and there is only one independent supersymmetry parameter. The θ's which are presently inert under supersymmetry will obtain their supersymmetry transformations after κ-gauge fixing.

The only spacetime scalars we have are the embedding coordinates $X^\mu(\sigma)$ for worldvolume directions. There are several options that one could consider for the X^μ:

1. We could assume that we are in the static gauge, i.e.

$$X^\mu(\sigma) = \delta_i^\mu\,\sigma^i\,, \tag{17}$$

from the beginning, so that the X^μ are absent. In this case there are no worldvolume reparametrisations, i.e. $\xi^i = 0$ in (14).

2. We could decide that the X^μ are in the singlet representation of the Yang-Mills group. The idea is that the n branes overlap, there is only one set of worldvolume coordinates, and the corresponding reparametrization group would be sufficient to gaugefix a singlet set of embedding coordinates.

3. We could choose the X^μ in the adjoint representation of Yang-Mills in analogy with transverse coordinates for $p < 9$. Here one thinks of starting with n separate branes where each has its own worldvolume and embedding coordinates. When the branes overlap the embedding coordinates "fill up" to form elements of the adjoint representation. Clearly this requires a different approach towards the world-volume reparametrisation invariance, which must then correspond to a sufficiently large symmetry group to gaugefix all these embedding functions.

We have investigated the first two possibilities in the non-abelian case, and we have found that only the first approach is consistent with the iterative procedure that we employ.

Our strategy will be to construct a kappa-symmetric action via an iterative procedure. We found that the flat background expressions of the abelian NS-NS and R-R fields cannot be generalized to the non-abelian case while remaining YM singlets. We therefore decided to make the most general Ansatz for the action and to require kappa-symmetry, order by order in F and up to terms quartic in the fermions. The iteration is obtained by expanding Γ and $\mathcal{T}$ in the variation (5) order by order in F:

$$\begin{aligned} \delta\mathcal{L} &= -\bar{\eta}\,(1-(\Gamma_0+\Gamma_1+\ldots))(\mathcal{T}_0+\mathcal{T}_1+\ldots) \\ &= -\bar{\eta}\,(\mathcal{T}_0-\Gamma_0\mathcal{T}_0+\mathcal{T}_1-\Gamma_1\mathcal{T}_0-\Gamma_0\mathcal{T}_1+\ldots)\,, \end{aligned} \tag{18}$$

where the subindices reflect the order in F. We completed this procedure up to variations of the action quadratic in F. Due to lack of space we give the result, both in its kappa-symmetric as well as in its gauge-fixed form, not here, but in a forth-coming publication.[25]

3. Comments

We have been able to embed a D=10 supersymmetric $U(n)$ YM theory (up to terms in the action quartic in F) into a kappa-symmetric system. In particular, we find fermionic terms of the form $[F^2,\bar{\chi}]\,\partial\chi$ which violate the symmetrized trace prescription.

We hope that our partial results will contribute to a better understanding of the non-abelian Born-Infeld action and perhaps may lead to the complete answer like in the abelian case. In this respect it is of interest to remember that the abelian expression (9) for $\Gamma(X,F)$ can be written as[10]

$$\Gamma(X,F) = e^{-a/2}\Gamma^{(0)}(X)e^{a/2}\,, \tag{19}$$

with

$$a = \tfrac{1}{2}Y_{ij}\Gamma^{ij}\sigma_3\,. \tag{20}$$

The matrix Y is related to the matrix F by a so-called "tan" relation defined in Ref. 10: F = "tan" Y. These results were obtained by looking to branes at angles.[26,27] Via T-duality these systems are related to branes with $F \neq 0$ as we discuss in this talk. It would be interesting to see whether a similar simple expression can be found for the nonabelian case

Another approach to find the complete answer could be to use the superembedding techniques developed in Refs. 28,29. Finally, yet another way to get the complete answer could be to extend to the non-abelian case the analysis of Ref. [30] where it was shown how the superworldvolume dynamics of superbranes can be obtained form nonlinear realizations.

We would like to end by mentioning a few important open issues.

(A) Can the results we obtained be generalized to a supersymmetric curved background? On the one hand the flat background expressions we find do not fit into general supergravity background field expressions that are YM singlets. We do not allow the supergravity fields to be in a nontrivial representations of the YM group. One possible scenario could be that the D-brane only couples to the U(1) part of the N=1 supergravity background fields and that the off-diagonal terms only involve worldvolume fields. The price to pay is that not all worldvolume fields are on the same footing whereas, at least after fixing the kappa-symmetries, they should be.[c]

(B) A somewhat related issue is: is there a natural (non-abelian) superspace geometry that describes the kappa-symmetric result? It is well-known that, to describe the abelian kappa-symmetric action, an important role is played by ordinary superspace geometry. In the non-abelian case we are dealing with many fermionic coordinates on which just a single N=2 supersymmetry is realized. Usually, introducing more fermionic coordinates, means going to an extended superspace with extended supersymmetries but this is not the case in the present situation.

(C) It would be interesting to see what happens when we T-dualize our results and obtain the worldvolume theory of non-abelian Dp–branes with $p < 9$. Our hope is that this will teach us something about the D-geometry which is involved in describing such systems.

(D) Finally, our results can be used to search for non-Abelian BPS configurations, i.e. configurations with nontrivial F-terms that take values outside the Cartan subalgebra.

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References

1. A recent review of Born-infeld theory can be found in hep-th/9908105, to appear in the Yuri Golfand memorial volume.
2. E.S. Fradkin and A.A. Tseytlin, *Phys. Lett.* **B163**, 123 (1985).
3. A. Abouelsaood, C.G. Callan, C.R. Nappi and S.A. Yost, *Nucl. Phys.* **B280**, 599 (1987).
4. E. Bergshoeff, M. Rakowski and E. Sezgin, *Phys. Lett.* **B185**, 371 (1987).
5. R.R. Metsaev and M.A. Rakhmanov, *Phys. Lett.* **B193**, 202 (1987).
6. S. Cecotti and S. Ferrara, *Phys. Lett.* **B187**, 335 (1987).
7. M. Aganagic, C. Popescu and J.H. Schwarz, *Nucl. Phys.* **B495**, 99 (1997) [hep-th/9612080].
8. M. Cederwall, A. von Gussich, B.E.W. Nilsson, P. Sundell and A. Westerberg, *Nucl. Phys.* **B490**, 179 (1997) [hep-th/96100148].
9. E. Bergshoeff and P.K. Townsend, *Nucl. Phys.* **B490**, 145 (1997) [hep-th/9611173].
10. E. Bergshoeff, R. Kallosh, T. Ortin and G. Papadopoulos, *Nucl. Phys.* **B502**, 149 (1997) [hep-th/9701127].
11. F. Denef, A. Sevrin and J. Troost, *Nucl. Phys.* **B581**, 135 (2000) [hep-th/0002180].
12. S.V. Ketov, *Phys. Lett.* **B491**, 207 (2000) [hep-th/0005265]; S.V. Ketov, *Class. Quant. Grav.* **17**, L91 (2000) [hep-th/0005126].
13. A. Refolli, N. Terzi and D. Zanon, *Phys. Lett.* **B486**, 337 (2000) [hep-th/0006067].
14. W.I. Taylor and M. Van Raamsdonk, *Nucl. Phys.* **B573**, 703 (2000) [hep-th/9910052].
15. R.C. Myers, *JHEP* **9912**, 022 (1999) [hep-th/9910053].
16. M.R. Douglas, *Class. Quant. Grav.* **17**, 1057 (2000) [hep-th/9910170].
17. J. de Boer, talk presented at the Euresco conference *Particle and Physics, Euroconference on Quantum Fields and Strings*, September 9-15 2000, Kolymbari, Crete, Greece.
18. E. Witten, *Nucl. Phys.* **B460**, 35 (1996) [hep-th/9510135].
19. D.J. Gross and E. Witten, *Nucl. Phys.* **B277**, 1 (1986); A.A. Tseytlin, *Nucl. Phys.* **B276**, 391 (1986); **B291**, 876 (1987).
20. D. Brecher and M.J. Perry, *Nucl. Phys.* **B527**, 121 (1998) [hep-th/9801127];
    K. Behrndt, proceedings of the XXIII Int. Symp. Ahrenshoop 1989, 174, Akademie der Wissenschaften der DDR; *Untersuchung der Weyl-Invarianz im Verallgemeinter σ-Modell für Offene Strings*, PhD thesis, Humboldt-Universität zu Berlin, 1990.
21. A.A. Tseytlin, *Nucl. Phys.* **B501**, 41 (1997) [hep-th/9701125].
22. P.C. Argyres and C.R. Nappi, *Nucl. Phys.* **B330**, 151 (1990).
23. A. Hashimoto and W. Taylor, *Nucl. Phys.* **B503**, 193 (1997) [hep-th/9703217].
24. P. Bain, proceedings of the Cargèse '99 Summer School, hep-th/9909154.
25. E. Bergshoeff, M. de Roo and A. Sevrin, in preparation.
26. M. Berkooz, M.R. Douglas and R.G. Leigh, *Nucl. Phys.* **B480**, 265 (1996) [hep-th/9606139].
27. V. Balasubramanian and R.G. Leigh, *Phys. Rev.* **D55**, 6415 (1997) [hep-th/9611165].

28. I.A. Bandos, D. Sorokin, M. Tonin, P. Pasti and D.V. Volkov, *Nucl. Phys.* **B446**, 79 (1995) [hep-th/9501113].
29. P.S. Howe, E. Sezgin and P.C. West, proceedings of the Volkov memorial conference, hep-th/9705093.
30. S. Bellucci, E. Ivanov and S. Krivonos, *Phys. Lett.* **B482**, 233 (2000) [hep-th/0003273].